

LAB QUALITY INTERNATIONAL **Eurachem**
A focus for analytical chemistry in Europe

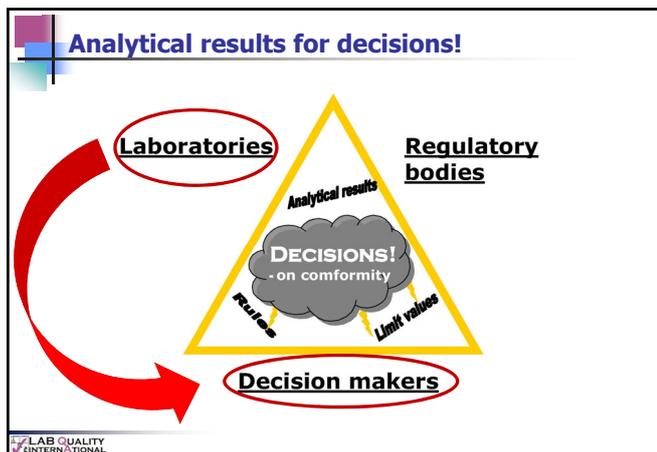
The use of Decision Rules

Presentation at
 A two-day training course
 ACCREDITATION OF ANALYTICAL, MICROBIOLOGICAL AND MEDICAL
 LABORATORIES - ISO/IEC 17025:2017 AND ISO 15189:2012

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Testing for Conformity Assessment

- ✓ Laboratories often do testing as part of a conformity assessment
 - Measuring whether a parameter is within tolerance limits
 - Testing whether content of a sample is in accordance with specifications ...etc.
- ✓ Conformity vs. Non-Conformity
 - Compliance vs. Non-Compliance
 - Acceptance vs. Rejection
 - Pass vs. Fail
- ✓ Decisions not always straightforward

The diagram shows a tolerance interval (from T_L to T_U) and an acceptance interval (from A_L to A_U). The acceptance interval is a subset of the tolerance interval. Below, a series of bell curves are shown with an upper limit line, illustrating how different distributions relate to the upper limit.

2017 version of ISO/IEC 17025 requirement

- ✓ In paragraph 7.1 on agreements with client:
 - When the customer *requests a statement of conformity to a specification or standard* for the test or calibration (e.g. pass/fail, in-tolerance/out-of-tolerance), the specification or standard and *the decision rule shall be clearly defined*. Unless inherent in the requested specification or standard, the decision rule selected shall be communicated to, and agreed with, the customer. [7.1.3]
- ✓ ..and in paragraph 7.8 on reporting of results:
 - When a statement of conformity to a specification or standard is provided, *the laboratory shall document the decision rule employed, taking into account the level of risk (such as false accept and false reject and statistical assumptions) associated with the decision rule employed, and apply the decision rule.* [7.8.6.1]

Risk of making wrong decisions

- ✓ Many decisions made on the basis of some tests and measurements
- ✓ BUT tests & measurements do not provide a 100% sure basis for making the correct decision
 - Tests/measurements only (!) made on a sample from the material/items for which the decision must be made.
 - There is a measurement uncertainty to the result of the test /measurement
 - The decision may be biased if somebody have a special interest in the outcome of the decision
- ✓ There will always be a RISK of making a wrong decision

Errors and risks

- ✓ **Errors** are always made during measurements and tests
 - Systematic errors ⇒ Biased results (...to be dealt with!)
 - Random errors ⇒ Measurement uncertainty
- ✓ These errors leads to **the risk of making errors** when the results are used as **basis for decisions**
 ... even two types of errors:
 - **Type I**: Deciding that something is NOT OK – when it really is OK.
 - given the probability (risk): **α**
 - **Type II**: Deciding something is OK – when it really was NOT
 - given the probability (risk): **β**

Actual situation	Decision	
	Accept H_0	Reject H_0
H_0 (True)	Correct decision	Type I Error
H_1 (False)	Type II Error	Correct decision

MU influences the interpretation of a result

When is the limit exceeded?

(a) (b) (c) (d) (e) (f) (g)

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Typical (simple) guidance

Positive compliance

Positive non-compliance

(a) (b) (c) (d) (e) (f) (g)

In doubt

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How does Measurement Uncertainty affect interpretation?

When is the limit exceeded?

(a) (b) (c) (d) (e) (f) (g)

In doubt

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Decision rules

✓ The key to the assessment of compliance is the concept of "**Decision rules**".

- These rules give a prescription for the acceptance or rejection of a product based on the
 - measurement result,
 - its uncertainty and
 - the specification limit or limits,...taking into account the acceptable level of the probability of making a wrong decision.

✓ And it has become a requirement in the 2017 version of ISO/IEC 17025!



What do we need for a decision ?

1. A **measurand** clearly specified
A specification of the **measurement object/test item** (part of measurand)
2. A **test result**
(normally assuming normal distribution of test results)
3. A **measurement uncertainty**
For an expanded uncertainty the *k* factor and the corresponding confidence level should be stated e.g. *k* = 2 for 95 % confidence
4. A **specification** giving **upper and/or lower limits**
5. A **decision rule**
This rule can decide to *take* or *not to take* measurement uncertainty into account
 - AND it can include the **risk** of making a wrong decision, which the involved parts are willing to take



Decision rule

✓ Definitions

- Documented rule that describes how **measurement uncertainty** will be accounted for with regard to accepting or rejecting an item, given a specified requirement and the result of a measurement
[Re. ISO Guide 98-4 = JCGM 106]
- A documented rule that describes how **measurement uncertainty** will be allocated with regard to accepting or rejecting a product according to its specification and the result of a measurement.
[Re. Eurachem Guide on Compliance assessment]
- Rule that describes how **measurement uncertainty** is accounted for when stating conformity with a specified requirement
[Re. ISO/IEC 17025:2017]



Simple decision rules

Decision rules may be very simple:

- A result implies non-compliance with an upper limit if the measured value exceeds the limit by the expanded uncertainty.
- A result equal to or above the upper limit implies non-compliance and a result below the limit implies compliance - provided that uncertainty is below a specified value. This is normally used where the **uncertainty** is so **small compared with the limit** that the risk of making a wrong decision is acceptable.
- BUT to use such a rule without specifying the maximum permitted value of the uncertainty would mean that **the probability (risk) of making a wrong decision would not be known.**

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More complicated rules

What about situation II and III?

- Depending on the actual situation the rules may include - e.g.
 - A request for additional measurement(s)
 - A manufactured (and tested) product must be compared with an alternative specification to decide on possible sale at a different price.

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Use of the statistical distribution

- Decision rule based on hypothesis $H_0: P(y \leq T_U) \geq (1-\alpha)$ - where α is the acceptable risk for false rejection
- A conformance probability P_C is calculated based on the normal cumulative distribution (Φ):

$$P_C = P(\eta \leq T_U) = \Phi((T_U - y)/u(y))$$
 (where η is a variable describing possible values of a measurand Y)

NOTE: Φ in Excel: NORM.DIST(T_U ; y ; $u(y)$; TRUE)

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Use of the statistical distribution - Example 1

- Consider a measurement estimate $y = 2,7$ mm with a standard uncertainty of $u(y) = 0,2$ mm, a single tolerance upper limit of $T_U = 3,0$ mm, and a specification of conformity $(1 - \alpha)$ of 0,95 (95 %), thus assuming a type I error $\alpha = 0,05$ (5 %).
- Normal distribution assumed.
- Decision rule:**
 Acceptance if the hypothesis $H_0: P(y \leq 3,0 \text{ mm}) \geq 0,95$ is true.
- Calculation of P_C :

$$P_C = \Phi((3,0 - 2,7) / 0,2) = \Phi(1,5) \Leftrightarrow$$

$$P_C = 0,933 \approx 93,3\%$$
(NORM.DIST(3,0; 2,7; 0,2; 0,2; TRUE) - or look up 0,15 in cum. norm. distr. table)
- As $93,3\% < 95\%$ the H_0 is rejected and the **the sample is declared Non-compliant** with the tolerance.

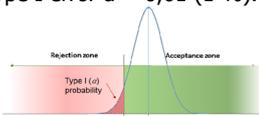
Having a risk of $\alpha = 6,7\%$ of false rejection

Use of the statistical distribution - Example 2

- Consider a measurement estimate $y = 0,012$ g with a standard uncertainty of $u(y) = 0,001$ g, a single tolerance lower limit of $T_L = 0,010$ g, and a specification of conformity $(1 - \alpha)$ of 0,99 (99 %) thus assuming a type I error $\alpha = 0,01$ (1 %).
- Decision rule:**
 Acceptance if the hypothesis $H_0: P(y \geq 0,010 \text{ g}) \geq 0,99$ is true.
- Calculation of P_C :

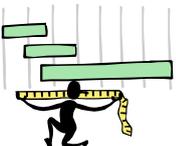
$$P_C = 1 - \Phi((0,01 - 0,012) / 0,001) = \Phi((0,012 - 0,01) / 0,001) = \Phi(2,0)$$

$$\Leftrightarrow P_C = 0,977 \approx 97,7\%$$
- As $97,7\% < 99\%$ the H_0 is rejected and the the sample is **declared Non-compliant** with the tolerance.
- Having a risk of $\alpha = 2,3\%$ of false rejection
(..and only 1% was acceptable; conforming if 5% had been acceptable)



Use of Acceptance/Rejection Zones

- On the basis of the Decision rules we can determine ...
 - An "**Acceptance zone**"
 - If the result lies here, the sample/product is declared **compliant**
 and
 - A "**Rejection zone**"
 - If the result lies here it is declared **non-compliant**.

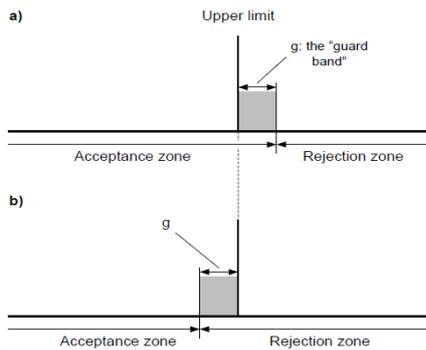


Use of "Guard Bands"

- ✓ Decision rule for non-compliance or rejection with **low risk of false rejection** (high confidence of correct rejection).
- ✓ A rejection zone can be defined as starting from the specification limit L plus an amount g - the **Guard band**. (see figure, case a)
- ✓ The value of g is chosen so that for a measurement result greater than or equal to L+ g the **probability of false rejection** is less than or equal to **α – the accepted risk**;
- ✓ The Guard Band, g, can also be chosen to provide **low risk of false acceptance** (case b)

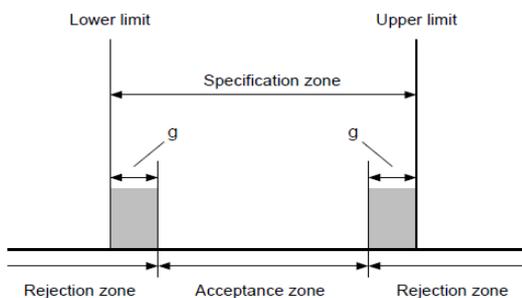
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Guard bands!!



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Double Guard bands!!



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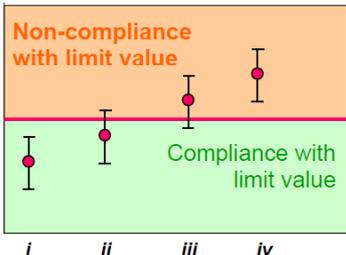
Establishing the Guard Band

- ✓ In general, g will be a **multiplum of the standard uncertainty u** .
 - For the case where the distribution of the likely values of the measurand is approximately normal, a value of $1.64u$ will give a risk, α , of 5% and a value of $2.33u$ implies a risk, α , of 1%.
 - I.e. the "one-tailed" t-factor on 5% or 1% level
- ✓ In some cases the decision rule may state the value of the multiplum to be used.
- ✓ In specific cases the guard band will **depend upon the acceptable risk-value α** and the **knowledge about the distribution** of the likely values of the measurand



Use of Guard Bands - Examples

- ✓ Based on the "classical picture":



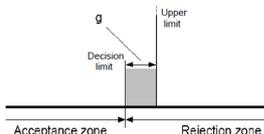
- ✓ Test results with expanded uncertainty in relation to an upper limit

Use of Guard Bands - Example 1

- ✓ Case *ii* in the figure with an upper limit
 - and a decision rule focusing on **correct acceptance**
 - Sludge from water purification plants can be used for soil improvement. One of the toxic metals that can be a problem is cadmium. The upper limit on the total cadmium in sludge is set to 2 mg/kg.
- ✓ The following requirements for decision in place:
 - **Measurand:** Mass fraction of cadmium, Cd, in a consignment delivered to a customer
 - **Analytical result:** Mass fraction (Cd) = 1.82 mg/kg
 - **Uncertainty:** $U = 0.20$ mg/kg, $k=2$ (95 %).
Standard uncertainty, $u = 0.10$ mg/kg. The uncertainty includes both sampling and analytical uncertainty
 - **Specification:** Upper permitted limit 2.0 mg/kg

Example 1 (cont'd)

- Decision rule: The decision limit is the mass fraction where it can be decided with a confidence level of approximately 95 % ($\alpha=0.05$) that the batch has a mass fraction below the upper limit.
- The guard band is calculated as $1.65 \cdot u = 0.165 \text{ mg/kg}$ (k value 1.65 for one tailed t value at 95 % confidence). The decision limit will be $2 - 0.165 = 1.84 \text{ mg/kg}$. All values below this value are in the acceptance zone. All values equal to or over are in the rejection zone.



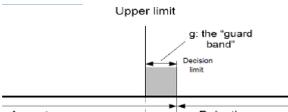
The sludge sample meets the compliance requirements.

Use of Guard Bands - Example 2

- Case *iii* in the figure with an upper limit - and a decision rule focusing on **correct rejection**
 - In law it is important not to punish an innocent person. The decision limit can be set to reduce the chance of this happening. This is an example from measurement of blood alcohol (EtOH) in a sample taken from a driver in Sweden who tested positive in a screening test.
- The following requirements for decision in place:
 - Measurand:** Mass fraction of total EtOH in a blood sample as delivered to the laboratory
 - Analytical result:** Mass fraction (EtOH) = 0.221 mg/g
 - Uncertainty:** $U = 0.013 \text{ mg/g}$, $k=2$ (95 %). Standard uncertainty, u , 0.0065. This uncertainty includes both sampling and analytical uncertainty.

Example 2 (cont'd)

- Regulation:** Upper permitted limit 0.200 mg/g
- Decision rule: The decision limit is the mass fraction above which it can be decided with a confidence level of approximately 99.9 % ($\alpha=0.001$) that the permitted limit has been truly exceeded.
- The guard band is calculated as $3.10u = 0.020 \text{ mg/g}$ (k value 3.10 for one tailed t-value at 99.9 % confidence). The decision limit will be $0.200 + 0.020 = 0.220 \text{ mg/g}$. All values below this value are in the acceptance zone (i.e. acceptance that the result does not justify a claim that the limit has been exceeded). All values equal to or over are in the rejection zone.



References

- ✓ ISO/IEC Guide 98-4, "Uncertainty of measurement - Part 4: Role of measurement uncertainty in conformity assessment", 1st ed., 2012 (= JCGM 106:2012)
- ✓ EUROLAB Technical Report No. 01/ 2017, "Decision rules applied to conformity assessment"
- ✓ Eurachem Leaflet, "Use of uncertainty information in compliance assessment" (2007)
- ✓ Eurachem Guide, "Use of uncertainty information in compliance assessment", 1st ed., 2007 (Under revision 2020)
