Future directions in measurement uncertainty evaluation

S L R Ellison
LGC Limited, Teddington, UK

Science
for a safer world

Introduction

- Existing approaches – a reminder
- A Bayesian approach to uncertainty evaluation
- Future guidance from JCGM

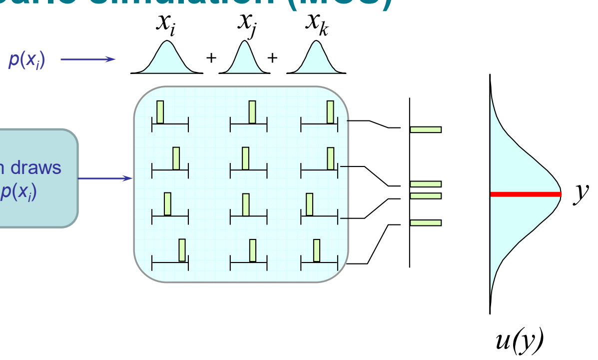
‘Law of propagation of uncertainty’ (LPU)

- Current GUM approach

$$u_i(y)=\sqrt{\sum_i\left(\frac{\partial y}{\partial x_i}\right)^2 u(x_i)^2}$$

- Limitations
  - Simple form assumes symmetry, small uncertainties, approximate linearity
  - Relies on simple form for *y* (must be differentiable)
  - Extensions allow for for correlation, non-linearity etc

Monte Carlo simulation (MCS)



- GUM Supplement 1
- Does not require differentiable form for *y*
- Allows for asymmetry, non-linearity
- Does not cope with constraints on *y*

Handling constraints on *y*:

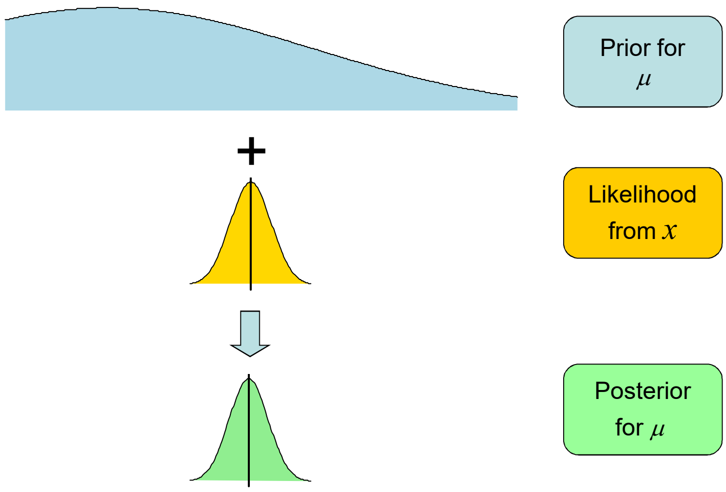
Bayesian methods

Bayes Theorem

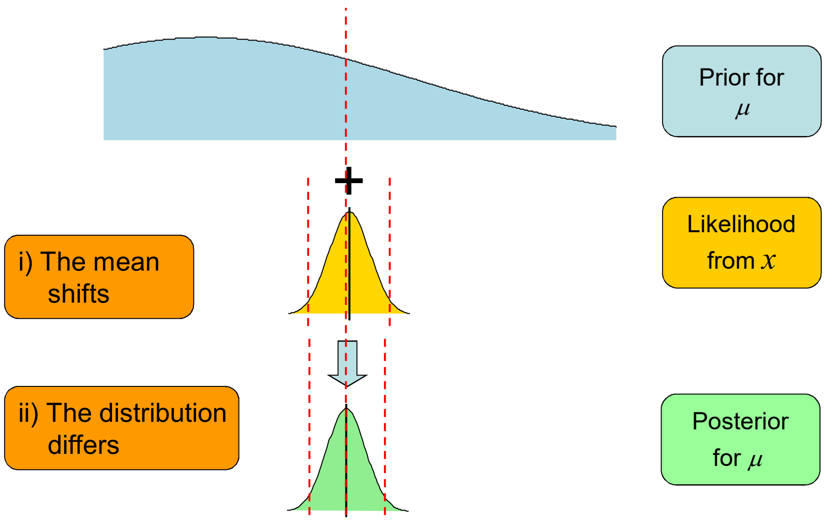
- Probability after a measurement depends on
  - The probability before the measurement
  - The 'strength' of evidence from the measurement

$$P(A|M) = P(A)\frac{P(M|A)}{P(M)}$$

Bayes applied to Measurement Uncertainty



Bayes applied to Measurement Uncertainty



Bayesian estimate using Markov Chain MC

MCS (Supplement 1)

- Samples from distributions for input quantities *x*
- Calculates *y*
- Generates a distribution for the value of the measurand if
  - Distribution of *x* does not depend on *y*
  - There are no prior constraints on *y*

Bayes/MCMC

- Starts from assumed distribution for μ
- Produces samples which reflect 'likelihood' of *y* given data *x*
- Always generates *a* distribution for the value of the measurand
- Depends somewhat on choice of prior

Bayes and measurement uncertainty: Avoiding controversy

Rule 1: The default: Use an uninformative prior

- typically wide Normal or Uniform

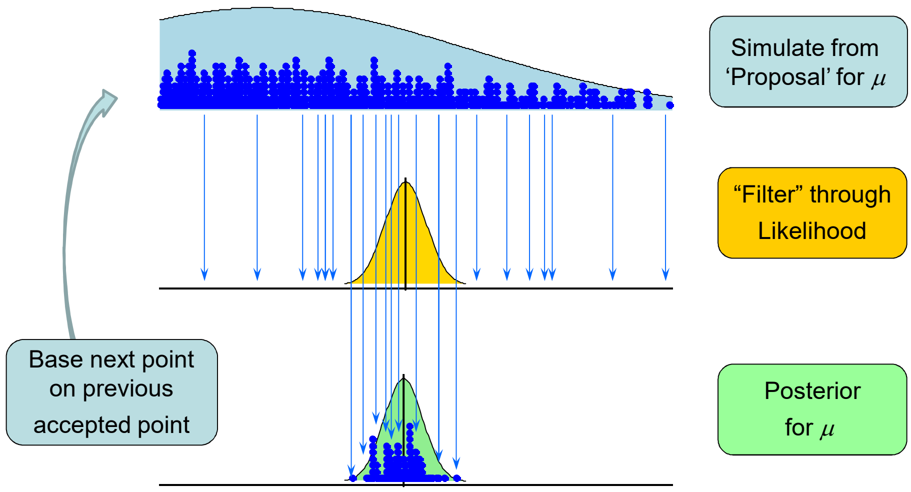
Rule 2: Ther[...]rs

- And so[...]pectedly informa[...]

Bayes theorem is most useful for *uncontroversial, informative* priors

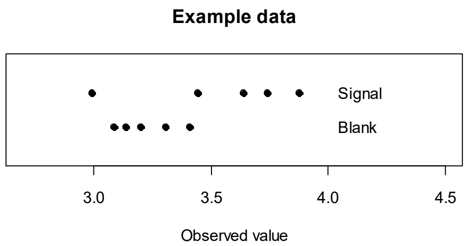
Rule 3: If an uninformative prior works for measurement uncertainty, there's probably an easier way

Bayes via Markov Chain Monte Carlo (MCMC)



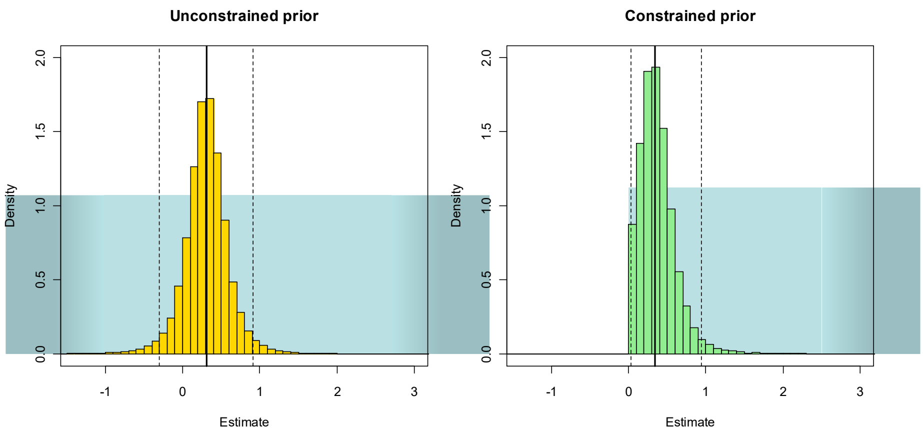
MCMC example

- *y* is a concentration calculated from a signal minus a blank value



- True concentration cannot be below zero

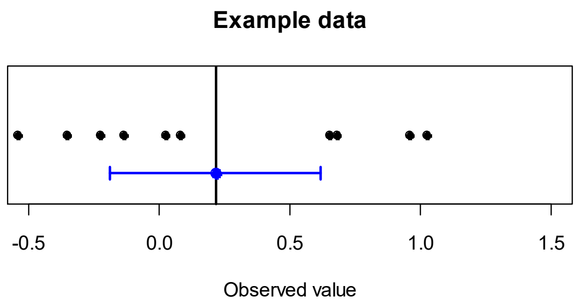
MCMC example - results



Uniform priors assumed for *y* and for both variances; error distributions assumed normal.

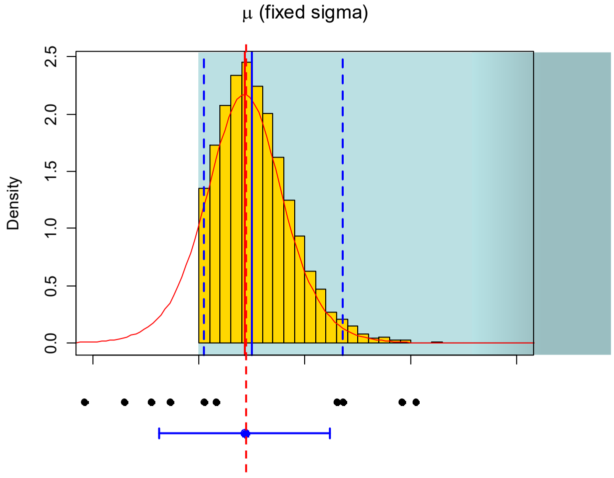
Calculations carried out using WinBUGS 1.4

MCMC example 2: Dispersion proportional to μ

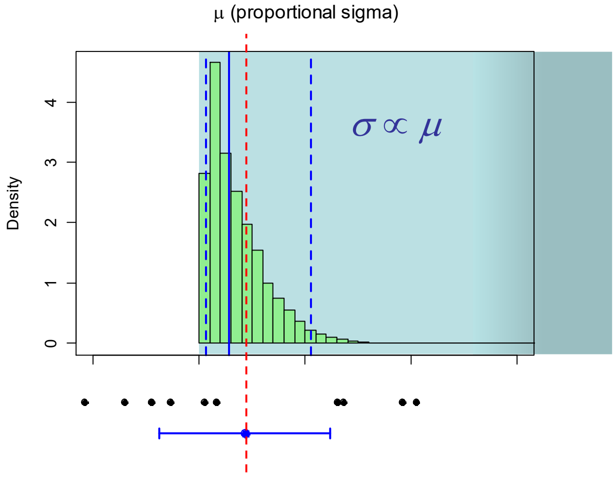


- Concentration: not below zero
- Common observation: standard deviation proportional to true value

MCMC results:
i) Fixed standard deviation



MCMC results
ii) Proportional standard deviation



The case for Bayesian methods

- Bayesian methods cope correctly with
  - Constraints on *y*
  - Distributions dependent on true value
- Bayes' theorem answers the right question
  - MCS: "Where could my next result be, if my result is the true value?"
  - Bayes: "Where could the true value be if this is my data?"
- BUT: Bayes is hard
  - Much more difficult – specialist software only
  - Choosing a 'prior distribution' is not simple
  - Interpretation needs care

Future JCGM guidance

JCGM

- Joint Committee for Guides in Metrology
  - Formed in 1997
- Members are international metrology and standards bodies:
  - BIPM, OIML, ISO, IEC, IUPAC, IUPAP, IFCC, ILAC*
- Responsible for guides on
  - Measurement uncertainty (WG1)
  - Terminology (the VIM) – WG2

*ILAC joined in 2005

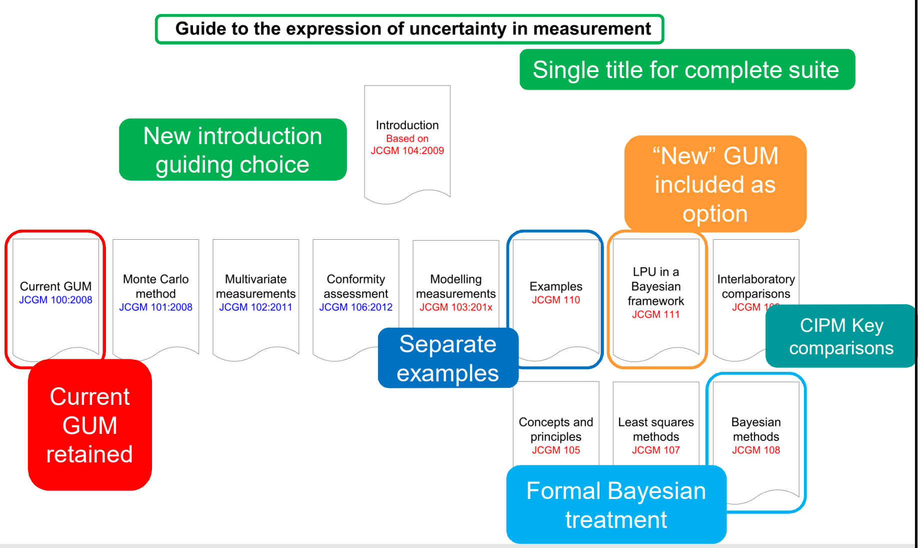
Existing JCGM guidance on MU

- JCGM 100:2008 Guide to the expression of Uncertainty in Measurement ("the GUM")
- JCGM 101:2008 Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method
- JCGM 102:2011 Supplement 2 to the "Guide to the expression of uncertainty in measurement" – Extension to any number of output quantities
- JCGM 104: An introduction to the "Guide to the expression of uncertainty in measurement" and related documents
- JCGM 106:2012 The role of measurement uncertainty in conformity assessment

Draft GUM replacement

- Bayesian basis
  - Used Bayesian posterior mean and variance for **input** quantities
  - LPU for small uncertainties
  - MCS (Supplement 1) for non-linear cases etc
- Issued for public comment in late 2014
- Substantial adverse comment from member bodies
- Development suspended

JCGM new direction



Summary

- JCGM see Bayesian reasoning as fundamental
- Existing guidance can be seen as special cases
  - GUM: Small uncertainties; Near normality; No ‘prior’ information
  - MCS: Non-normal distributions; No ‘prior’ information
- Future JCGM guidance is ‘modular’
  - Different treatments for different circumstances