Measurement Uncertainty-Small, Medium & Large
How to calculate the expanded uncertainty

A Williams

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How to calculate the expanded uncertainty

- Where small is < than about 15%

Measurement Uncertainty-Small, Medium & Large

- Where small is < than about 15%
- Where medium is < than about 50%

Measurement Uncertainty-Small, Medium & Large

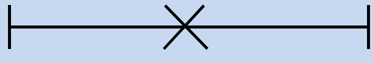
- Where small is < than about 15%
- Where medium is < than about 50%
- And large is > than 50%

Measurement Uncertainty-Small, Medium & Large

- When you see a statement of a result as *123± 1.2* or as 123± 1%, what comes to mind?

Measurement Uncertainty-Small, Medium & Large

- When you see a statement of a result as $x \pm ku$ or *123± 1.2* what comes to mind?
- Is it just a range of about 95% of the values of the measurand?



Measurement Uncertainty-Small, Medium & Large

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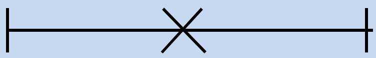
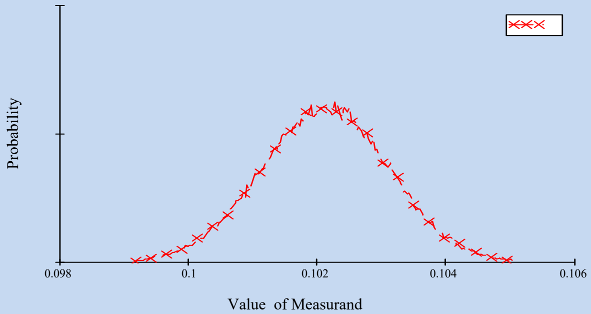
- Is it just a range of about 95% of the values of the measurand?

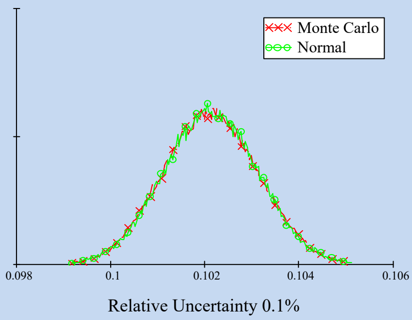


- Or is it a distribution of these values



Measurement Uncertainty-Small, Medium & Large

- To calculate the expanded uncertainty this distribution is needed
- According to GUM this distribution is most frequently Normal
- This was confirmed using the Monte Carlo method (1) applied to the data from example A2 in the Eurachem guide



Measurement Uncertainty-Small, Medium & Large

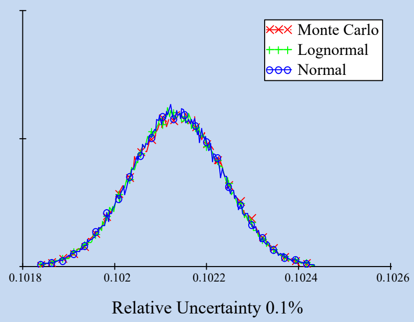
- However, the distribution could also be lognormal, if the model equation for calculating the value of the measurand consists of products of positive quantities, as is the case for Example A 2 in the Eurachem Uncertainty Guide where

$$c_{NaOH} = \frac{1000 \cdot (m1_{KHP} - m2_{KHP}) \cdot P_{KHP} \cdot Rep}{(M_{C8} + M_{H5} + M_{O4} + M_K) \cdot V_T \cdot T} \qquad [\mathrm{mol\,l^{-1}}]$$

- This is also the case for many analytical determinations..
- So what is the distribution, normal or lognormal?

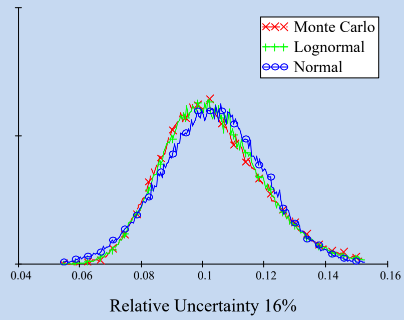
Measurement Uncertainty-Small,

- Monte Carlo calculations, again using the data from example A2, show that for small values of the relative uncertainty both distributions are the same!



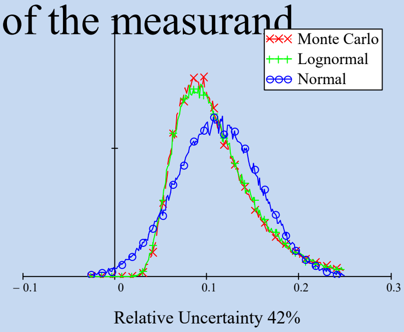
Measurement Uncertainty-Small, Medium

- However, a small difference appears at values of the relative uncertainty of 16%



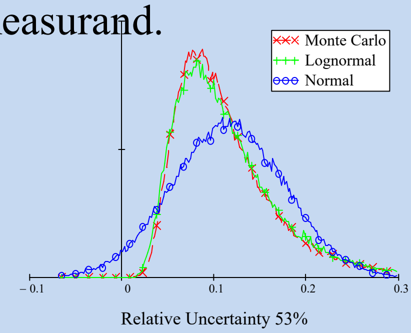
Measurement Uncertainty-Small, Medium & Large

- At larger values of the relative uncertainty, the two distributions differ significantly
- The lognormal agrees the Monte Carlo technique.
- The normal doesn't and also has a small probability of negative values of the measurand



Measurement Uncertainty-Small, Medium & Large

- At even larger values of the relative uncertainty the two distributions differ more.
- The lognormal agrees the Monte Carlo technique.
- The normal distribution has a larger probability of negative values of the measurand.



Measurement Uncertainty-Small, Medium & Large

- Therefore the distribution of the values attributable to the measurand is lognormal not normal.
- So how do we calculate the expanded uncertainty?

Measurement Uncertainty-Small

- For small relative uncertainties < 15% no change
- Both lognormal and normal applies.
- Therefore $x \pm ku$ can still be used

Measurement Uncertainty-Small, Medium

- For medium size relative uncertainties <40%
- Utilise the uncertainty factor UF as proposed by Ellison & Ramsey (2)
- The upper limit of expanded uncertainty is the mean multiplied by UF and lower limit is the mean divided by UF.

Measurement Uncertainty-Small and Medium

- For medium size relative uncertainties <40%
- The UF is $e^{ku_{rel}}$ and the mean is $\frac{\bar{x}}{\sqrt{1+u_{rel}^2}}$
- Therefore the limits of the expanded uncertainty are
- $U_{upper} = \frac{\bar{x}}{\sqrt{1+u_{rel}^2}} e^{ku_{rel}}$ and $U_{lower} = \frac{\bar{x}}{\sqrt{1+u_{rel}^2}} e^{-ku_{rel}}$

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- constant u_{rel} leads to constant expanded relative uncertainty

Measurement Uncertainty-Small, Medium & Large

- For large size relative uncertainties >40%
- The UF is $e^{k\sqrt{\ln(1+u_{rel}^2}}$ and the mean is still $\frac{\bar{x}}{\sqrt{1+u_{rel}^2}}$
- $U_{upper} = \frac{\bar{x}}{\sqrt{1+u_{rel}^2}} e^{k\sqrt{\ln(1+u_{rel}^2}}$ and

  $U_{lower} = \frac{\bar{x}}{\sqrt{1+u_{rel}^2}} e^{-k\sqrt{\ln(1+u_{rel}^2}}$

Measurement Uncertainty-Small, Medium & Large

- For large size relative uncertainties >40%
- Time to use software
- Most statistical packages include an inverse lognormal function, with input parameter *p, μ* and *σ* where
- $p = 0.023$ for the lower limit and 0.977 for the upper, for $k = 2$
- and $\mu = \ln(\frac{\bar{x}}{\sqrt{1+u_{rel}^2}})$
- and $\sigma = \sqrt{\ln(1 + u_{rel}^2)}$

Measurement Uncertainty-Summary

- Why do normal and lognormal agree for u_{rel} <15% ?
- We have seen that UF is $e^{ku_{rel}}$ and the mean is $\frac{\bar{x}}{\sqrt{1+u_{rel}^2}}$
- For small u_{rel} we can neglect u_{rel}^2 and $e^{ku_{rel}} \approx 1 + k\,u_{rel}$
- The confidence interval limits are $\bar{x}\,(1+ k\,u_{rel})$ and $\bar{x}\,(1- k\,u_{rel})$
- Which are $\bar{x} + k\,u$ and $\bar{x} - k\,u$ QED

Measurement Uncertainty-Summary

- For u_{rel} <15% use $x \pm ku$
- For u_{rel} < 40% use $UF = e^{ku_{rel}}$ and mean= $\frac{\bar{x}}{\sqrt{1+u_{rel}^2}}$
- For u_{rel} >40% use $UF = e^{k\sqrt{\ln(1+u_{rel}^2)}}$ and mean= $\frac{\bar{x}}{\sqrt{1+u_{rel}^2}}$
- Or use an “inverse lognormal” function

Measurement Uncertainty-References

1. Propagation of distributions using a Monte Carlo method, JCGM 101:2008
2. Ramsey M.H. Ellison S.L.R (2015) Uncertainty Factor: an alternative way to express measurement uncertainty in chemical measurement. Accreditation and Quality Assurance. 20, 2,153-155.doi:10.1007/s00769-015-1115-6

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