



# Values and uncertainties near the detection limit

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*The preparation of this paper was supported by Department for Business, Energy & Industrial Strategy  
under the National Measurement System Chemical and Biological Measurement Programme*



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# The Edge of Reason

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## Introduction

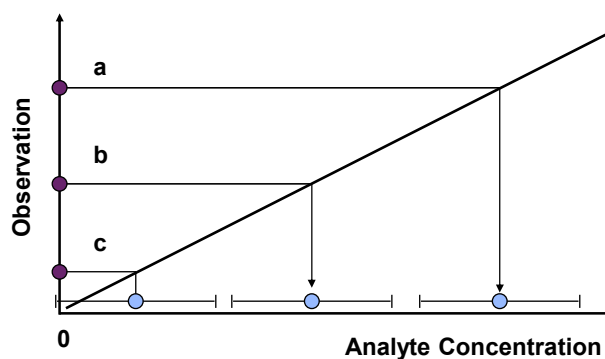


- The detection limit – a brief history
- Measurement Uncertainty near the detection limit



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## Detection limits

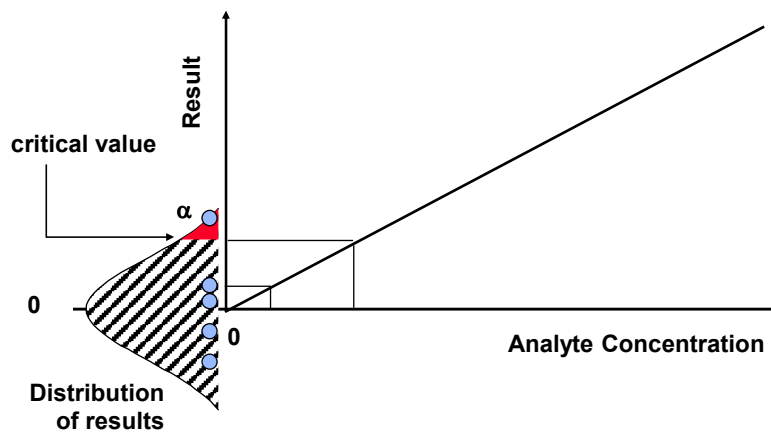


—●—  
Calculated  
concentration with  
uncertainty



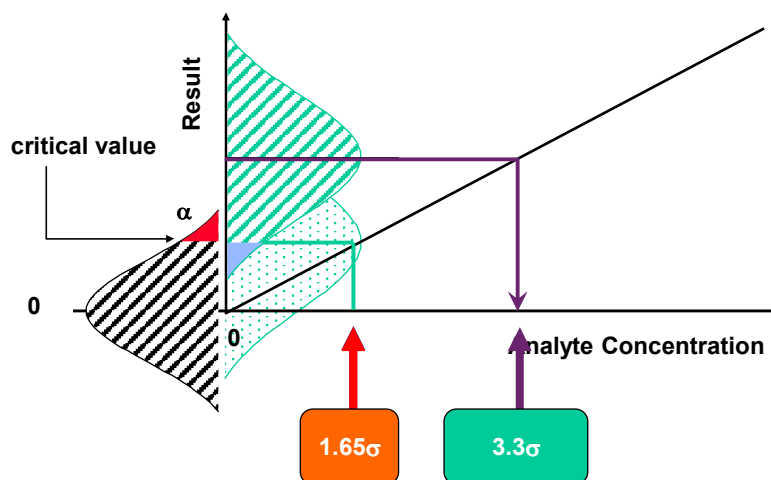
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## The detection limit: Part 1



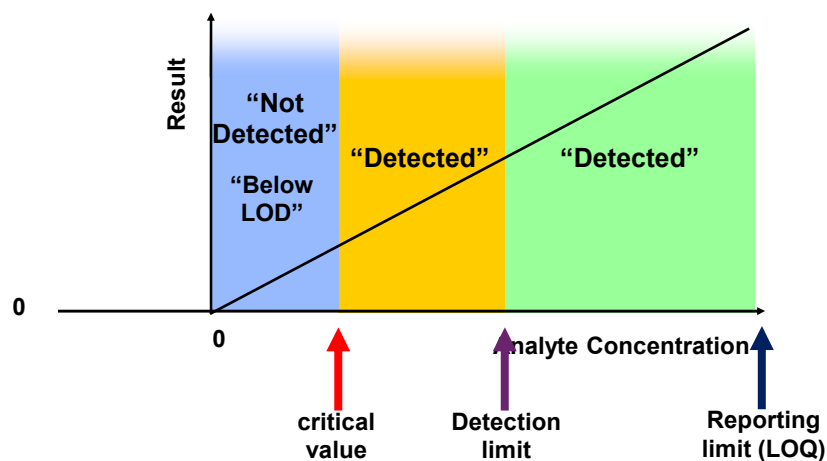
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## The detection limit: Part 2



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## The detection limit: Part 3



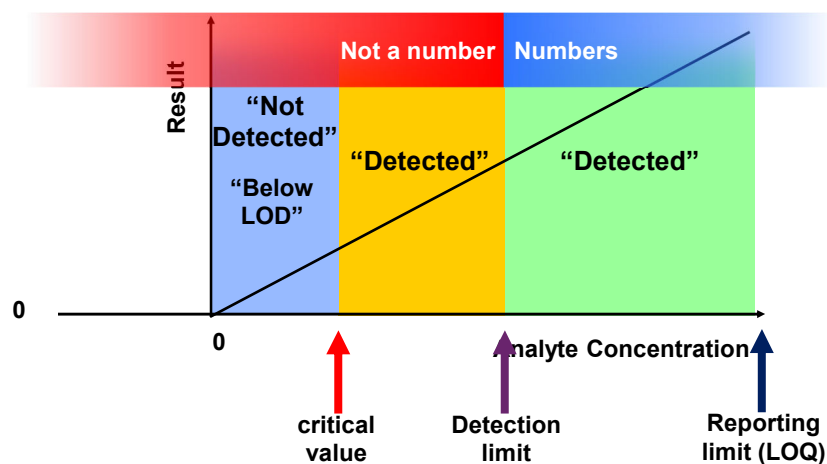
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**Results above the  
critical value and  
BELOW THE LOD  
are POSITIVE**



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## The detection limit: Common (mis)interpretations



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**All numerical results  
are valid estimates –  
even those  
below the LOD**



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## Using sub-LOD data



- “Less than” cannot\* be averaged
- “Less than” cannot\* be scored in PT
- “Less than” cannot be added
  - except to form a bigger “less than”
- ... or subtracted.
- ... but “-0.2 mg/kg  $\pm$  0.3 ” can...

Report the raw observation and uncertainty if at all possible



\*Maximum likelihood and approaches excluded

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## And there shall be...

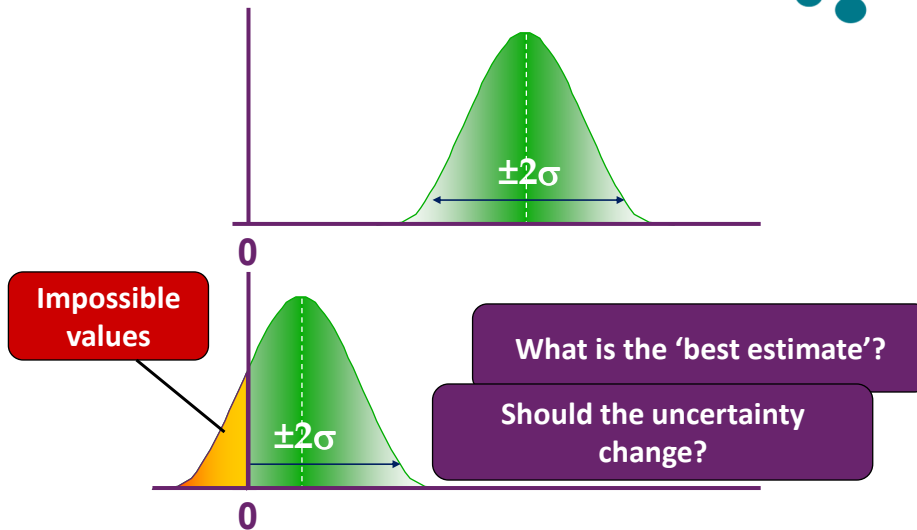


Measurement  
Uncertainty



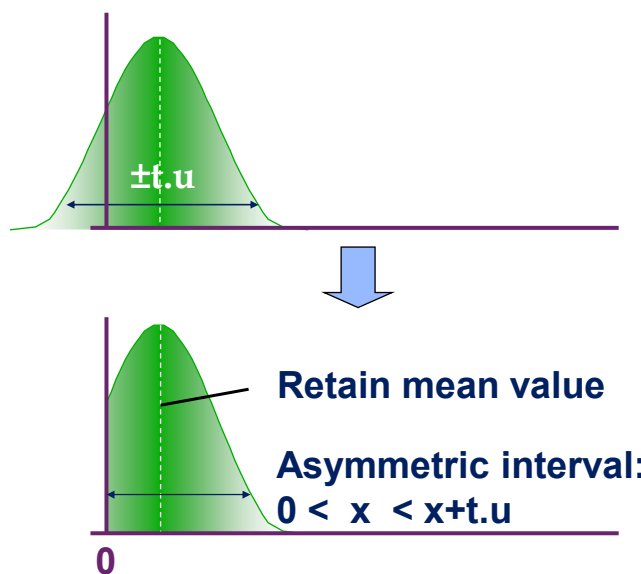
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# Uncertainty near zero/100%



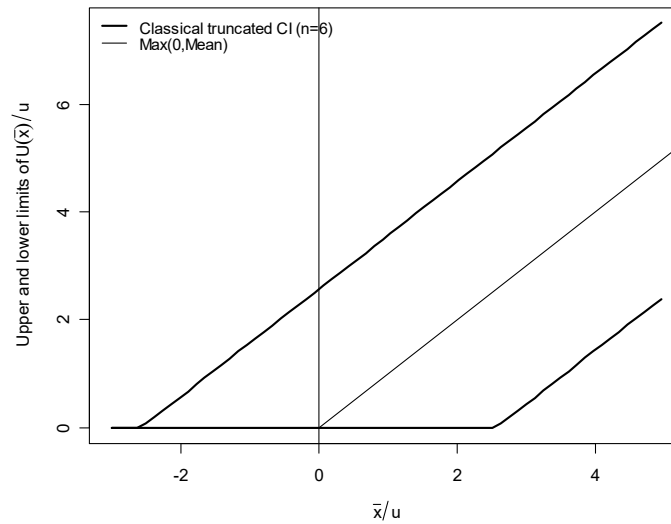
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# Truncation provides accurate coverage



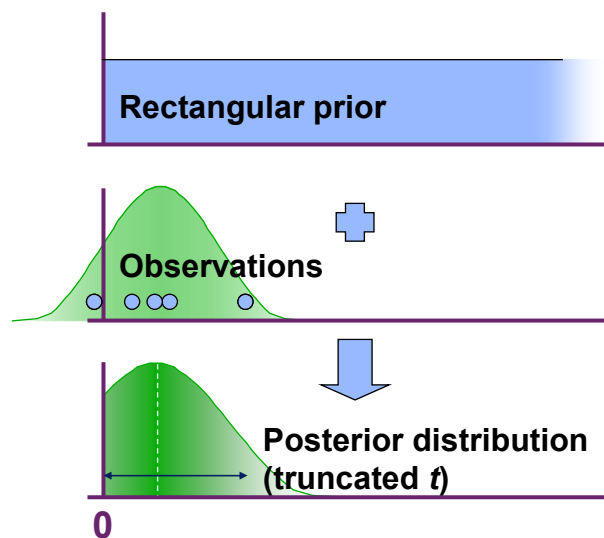
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## Truncated interval near zero



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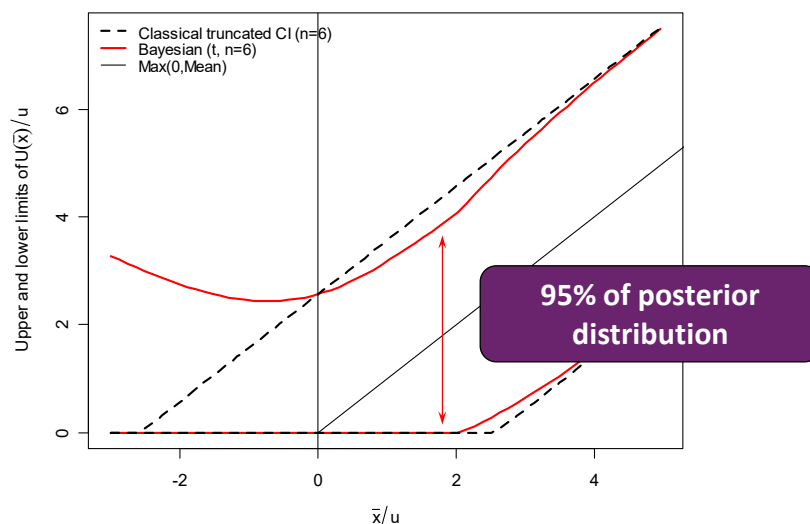
## Bayesian approach



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## Bayesian interval



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## Reporting measurement uncertainty on values near zero



- **Truncated interval retains exact coverage properties**
  - Standard uncertainty unchanged
  - Minimally biased mean
  - Convergence to zero width implies probable measurement failure
- **Bayesian interval more general but more complex to calculate**
- **Essential to truncate AFTER ALL OTHER CALCULATIONS**
  - Truncating interim values leads to increased bias

*“Handling undetected and low-level components in purity determination”.*  
*S Cowen, S L R Ellison, Accred. Qual. Assur. 12, 323-328 (2007)*



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“Less than LOD”  
does NOT mean  
“invalid result”



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## Can MU replace LOD?



### Advantages

- No arbitrary cut-off
  - Everything is a number
- Decision could be specific to the particular test item
  - Uncertainty for *that* result

### Challenges

- MU does not give information about the measurement procedure
- MU is complicated near zero
  - Traditional MU limits may include zero
  - Many useful Bayesian approaches could exclude zero
- MU should be about the test item
  - But often isn't
- An MU interval is two-sided
  - A detection decision is one-sided



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# Measurement uncertainty is not the same as detection capability



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## Conclusions and recommendations



- **Detection limits are based on statistical reasoning**
- **Detection limits are not decision limits**
- **Reporting limits are beyond the limit**
  
- **Report raw values if you can**
- **Reasonable uncertainty intervals are possible**
  - and may even replace 'limits'?
  
- **MU and detection capability give different information**
  - We may continue to need both, for different purposes



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